Admission Exam PPG-EM – 2024/2º sem

Candidate Name:	
ID:	
Date:	
Signature:	

Instructions

1) The exam consists of 10 questions, and the candidate must choose 5 questions to solve. In case the candidate solves a greater number of questions, only the first 5 will be considered;

2) All questions have the same value (2.0 points for each question);

3) It is not allowed to consult any type of material;

4) The use of simple (non-programmable) electronic calculators is permitted;

5) All answers must be justified;

6) The duration of the exam is 3 hours.

For the exclusive use of examiners

QUESTIONS SCORES									
Q1		Q3		Q5		Q7		Q9	
Q2		Q4		Q6		Q8		Q10	

TOTAL SCORE	

Graduate Program in Mechanical Engineering
São Carlos School of Engineering - University of São Paulo
Av. Trabalhador São-Carlense, 400, São Carlos, SP, 13566-590
Tel: 16 3373 9401

São Carlos School of Engineering - University of São Paulo
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QUESTION 1: (Linear Algebra)

Consider matrix A:

$$A = \begin{bmatrix} 3 & -8 & 8 \\ 0 & 4 & -1 \\ 0 & 5 & -2 \end{bmatrix}.$$

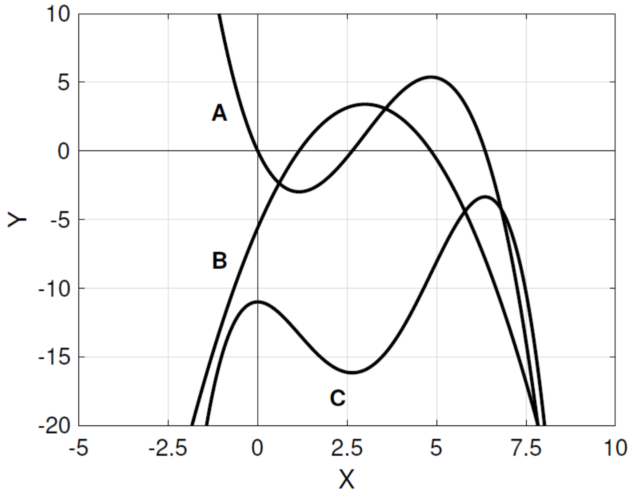
a) Knowing that one of the eigenvalues of matrix A is $\lambda_1 = 3$, determine the other eigenvalues λ_2 e λ_3.

b) Knowing that $|A| = -(n^2 + m^3)$, determine the values of n and m, where $n, m \in \mathbb{N}^*$.

Justify your answer.

QUESTION 2: (Differential and Integral Calculus)

a) The graph below presents three curves identified by the letters **A**, **B**, and **C**. Identify which curve is $f(x)$, which curve is $f'(x) = \frac{df}{dx}$, and which curve is $f''(x) = \frac{d^2f}{dx^2}$.



b) The functions $y_1 = x^2$ and $y_2 = 4 - x^2$ delimit a region in the space $x - y$. Calculate the area of the region delimited by these functions.

Justify your answer.

QUESTION 3: (Computation)

Considering the code below:

```c
#include <stdio.h>

void swap(int *v, int a, int b)
{
    int tmp = v[a];
    v[a] = v[b];
    v[b] = tmp;
}

void heapify(int *v, int n, int i)
{
    int ref = i;
    int left = 2 * i + 1;
    int right = 2 * i + 2;

    if (left < n && v[left] > v[ref])
        ref = left;

    if (right < n && v[right] > v[ref])
        ref = right;

    if (ref != i)
    {
        swap(v, i, ref);
        heapify(v, n, ref);
    }
}

void heapSort(int *v, int n)
{
    for (int i = n / 2 - 1; i >= 0; i--)
        heapify(v, n, i);

    for (int i = n - 1; i > 0; i--)
    {
        swap(v, 0, i);
        heapify(v, i, 0);
    }
}

void printvec(int *v, int n)
{
    for (int i = 0; i < n; i++)
    {
        printf("%d ", v[i]);
    }
    printf("\n");
}

int main()
{
    int arr[] = {10, 14, 42, 5, 4, 3, 2, 1, 8};
    int n = sizeof(arr) / sizeof(arr[0]);
    heapSort(arr, n);
    printf("Sorted array is:\n");
    printvec(arr, n);
}
```

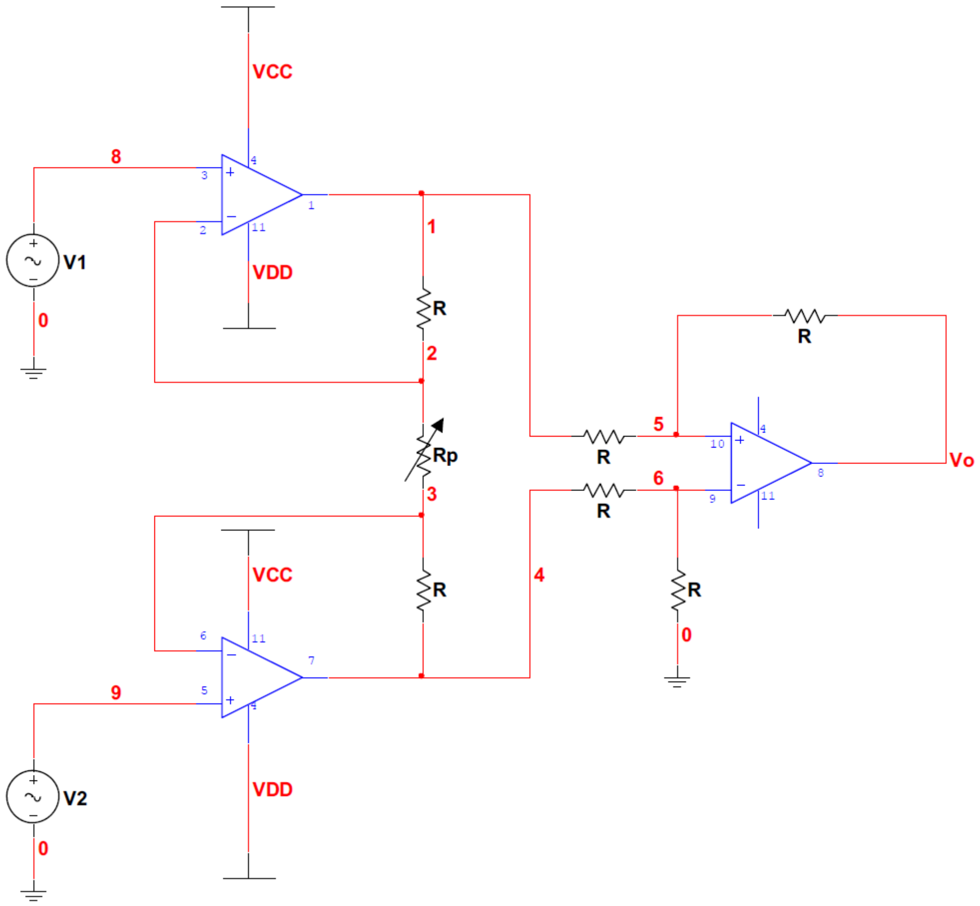
a) What is printed on the screen when the main function is executed?

b) What modifications need to be made to reverse the sorting order?

Justify your answer.

QUESTION 4: (Electronics)

The instrumentation amplifier, shown in the figure below, provides an output voltage Vo based on the difference between two inputs V1 and V2, multiplied by a gain factor. The gain can be tuned by using the potentiometer Rp.



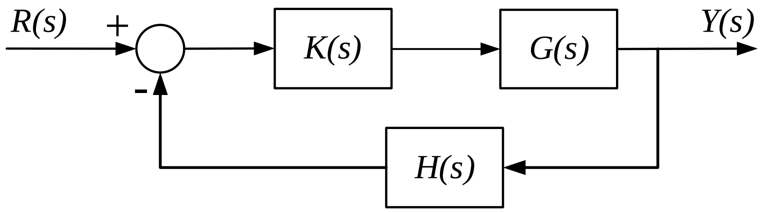
a) Obtain the expression for the circuit's input impedance, as seen by the voltage sources V1 and V2, considering the operational amplifiers as ideal devices.

b) Obtain the circuit's output voltage Vo with R = 5 kΩ and Rp = 500 Ω, as a function of the difference between the input voltages V1 – V2.

Justify your answer.

QUESTION 5: (Control Systems)

Considering the figure below:



a) Find the closed-loop transfer function *Y(s)/R(s)*.

b) Find the range of gains K that make the system stable.

Data: $K(s) = \frac{K}{s}$, $G(s) = \frac{1}{s+1}$ e $H(s) = 1$.

Justify your answer.

QUESTION 6: (Materials)

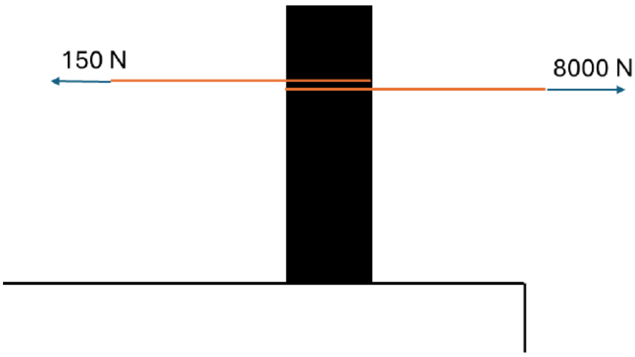
a) What is the difference between engineering stress and true stress in a tensile test?

b) Tensile testing is not appropriate for hard and brittle materials such as ceramics. What is the test commonly used to determine the strength properties of such materials?

Justify your answer.

QUESTION 7: (General Mechanics)

A rope is wound with two turns around a support. The tension at one end of the rope is 8,000 N, and on the other side, this force is balanced by a force of 150 N on the verge of slipping.



Determine:

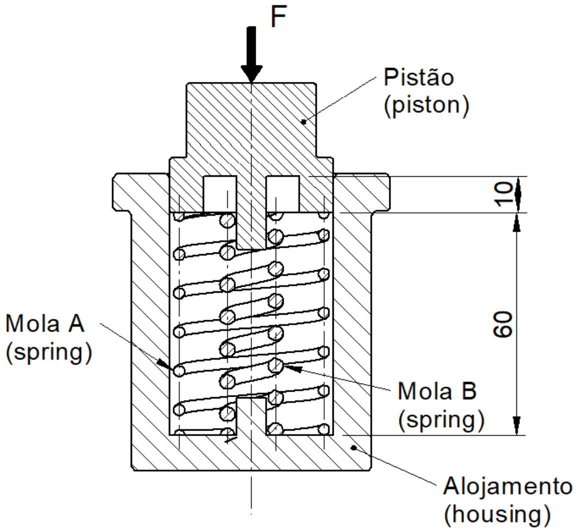
a) The coefficient of friction between the rope and the support.

b) The force that would balance the 8,000 N load if the rope were wound with three turns around the support.

Justify your answer.

QUESTION 8: (Solid Mechanics)

A two-stage load application system is composed of two coaxial springs as shown in the sectional figure to the side. The two springs have the same initial length L = 60mm (unloaded) and due to the expected deformation they are within the Hookean elastic regime.



Considering the rigid housing tube and piston:

a) What is the force discharged on spring A.

b) What is the total displacement of the piston when a force of 160N is applied?

Consider the spring constant A equal to 2 N/mm, the spring constant B equal to 5 N/mm and the displacement at the beginning of spring B actuation equal to 10 mm.

Justify your answer.

QUESTION 9: (Thermodynamics)

Carbon monoxide flows steadily through a thermal machine, entering at 170 kPa and 17 °C (u = 215.1 kJ/kg, h = 301.2 kJ/kg), and leaving at 90 kPa e -7 °C (u = 197.2 kJ/kg, h = 276.2 kJ/kg). The inlet velocity is negligible. The gas is discharged in a duct with a cross section of 0.0070 m^2 and at a velocity of 50 m/s. During the passage of CO through the machine, heat is transferred to the gas at a rate of 1.2 kJ/s.

Determine:

a) The power supplied by the thermal machine, in kW.

b) The mass flow rate used in the described operational condition, in kg/s.

Assume that CO behaves like an ideal gas with R = 297.0 J/(kg*K).

Formulae:
Mass conservation:

$$\frac{\mathrm{d}m_{\mathrm{vc}}}{\mathrm{dt}} = \sum_e \dot{m}_e - \sum_s \dot{m}_s$$

Energy conservation:

$$\dot{Q}_{\mathrm{vc}} = \frac{\mathrm{dE_{vc}}}{\mathrm{dt}} + \sum_s \dot{m}_s \left(h_s + \frac{V_s^2}{2} + \mathrm{g}z_s\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + \mathrm{g}z_e\right) + \dot{W}_{\mathrm{vc}}$$

Equation of state for an ideal gas: $Pv = \mathrm{RT}$

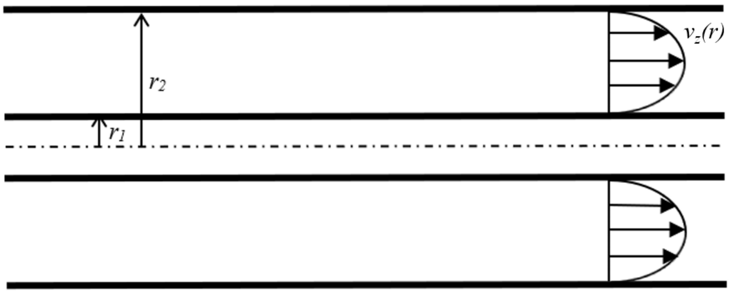
Where: $\dot{W}$ – Power [W]; E – Total energy [J]; $\dot{Q}$ – Heat Transfer Rate [W]; $\dot{m}$ – Mass flow rate [kg/s]; m – mass [kg]; T – temperature [K]; t – time [s]; g – gravitational acceleration [m/s^2]; z – height [m]; V – fluid velocity [m/s]; u - specific internal energy [J/kg]; h - specific enthalpy [J/kg]; v– specific volume [m^3/kg]; P – Thermodynamic pressure [Pa]; R – Gas constant (CO) [J/(kg*K)].

Subscripts: vc – control volume, s – outlet, e – inlet.

Justify your answer.

QUESTION 10: (Fluid Mechanics)

From the Navier-Stokes Equation, obtain an ordinary differential equation (ODE) for the distribution of velocities, $v_z(r)$, of the flow between horizontal and concentric tubes (annular duct). Highlight the simplifying assumptions and point out boundary conditions. It is not necessary to solve the ODE.



Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=$$

$$=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$$

Justify your answer.